Modelling in Curricula across the World

Panel discussion
ICTMA-17, Nottingham
July 24, 2015
Our journey:

- Japan (Monday)
- USA
- Netherlands & Norway
- Mexico
- Brazil
- Australia
- South Africa

Modelling in Curricula across the World
Questions to the panelists:

- What is the role of modelling in the curriculum of your country (perhaps including other, e.g. neighbouring, countries if appropriate), including assessment?

- Are there big discrepancies to actual classroom practice?

- What role - if any - has the international modelling community played in the developments in your country, and which role could it play?
Structure of the panel discussion:

- Introduction (3 min)
- Each panelist: 8 min statements plus 2 min questions from the audience (50 min)
- Plenary discussion with the audience (25 min)
- Each panelist: 2 min final statements (10 min)
- Closing remarks (2 min)
The Case of USA
The Role of Modeling in the United States Curriculum

Helen M. Doerr
Syracuse University
The Standards Documents

- NCTM 1989 Curriculum and Evaluations Standards for School Mathematics
- NCTM 2000 Principles and Standards for School Mathematics
- The Common Core State Standards for Mathematics 2010
The 1989 standards

- Modeling was part of the “problem solving” standard
In the 1989 standards

- One illustration of modeling was given in the document
- About the fairness of a two-player coin toss game
- Validation was suggested by playing the game or by computer simulation
Following the 1989 Standards

• Substantial curriculum development effort funded by the National Science Foundation

• Resulted in curriculum materials to meet these new standards at all grade levels
The 2000 Standards

- Modeling seems to disappear
- It is not mentioned in the “problem solving” standard
- BUT students are expected to “formulate, approach and solve problems”
- “problems that occur in real settings do not often arrive neatly packaged”
The 2000 Standards

• The “representation” standard states that all students should be able to “use representations to model and interpret physical, social, and mathematical phenomena”

• “recognize common mathematical structure across contexts”

• “create and interpret models” from a range of contexts
Curricular Materials

- Curricular materials continued to develop and became more widely adopted

- Two major projects:
  - CMP - Connected Mathematics Project for the middle grades
  - Core-Plus Mathematics Project for four years of high school
Political Events

• The so-called “math wars”
• “No Child Left Behind” (2001) – assessment to improve national outcomes
• The “Race to the Top” (2009) for “rigorous and challenging standards and assessments”
The Common Core State Standards in Mathematics

- The adoption of national standards by 45 states
- Modeling is a content standard in high school
- Modeling is a practice standard K-12
Modeling in the Common Core State Standards

- Compute means to analyze, perform operations on relationships, and draw conclusions.
Striking resemblance

Burkhardt (2007)
Common Core State Standards

• Sparked new interest in modeling As a practice:
  • “can apply the mathematics they know to solve problems arising in everyday life, society and the workplace”
  • make assumptions, identify quantities, draw conclusions, interpret results in context
Common Core State Standards

As content in high school:

• To analyze empirical situations, to understand them better, and to improve decisions

• Physical, economic, public policy, social, and everyday situations

• Technology for varying assumptions, exploring consequences, and comparing predictions with data
Common Core State Standards

- Assessments are being put in place
- Very little curriculum available, much is being “modified”
- Common Core Standards are becoming somewhat more controversial
The Cases of Mexico and Brazil
Modelling in the curriculum in Latinoamerica

Two cases: Mexico and Brazil

Ángeles Domíngues, Monterrey
Mathematics Standards (1-6 grade)

They consist of:

1. Number Sense and Algebraic Thinking
2. Form, space and measurement
3. Information management
4. Attitude towards the study of mathematics
Number Sense and Algebraic Thinking

Number sense and algebraic thinking refers to the most relevant late study of arithmetic and algebra:

- The **modeling situations** using the arithmetic language.
- The exploration of **arithmetic properties that will be generalized** with algebra in grades 7th to 9th and on.
- The **variety of ways to represent and perform** computations.
Role of modelling in the curriculum

At the end of grade 9th, students are responsible for construct new knowledge from their previous knowledge, which implies:

• To develop and validate conjectures.
• To ask questions.
• To communicate, analyze and interpret solutions and procedures.
• To argue the validity of procedures and results.
• To find different ways to solve problems.
• To use different strategies and techniques efficiently.
Teaching strategies

It is suggested for the study of mathematics, to use **sequences of problematic situations** to arouse the interest of students and invite them to **reflect**, to **find different ways to solve problems** and **make arguments that validate results**. At the same time, the situations should involve precisely the knowledge and skills to be develop.
Big discrepancies to actual classroom practice

• Tremendous gap between what it is stated on paper (mathematics reform) and practice (in the classroom).
Role of the modelling community

- Some efforts on bringing modelling into the classrooms

- Few research group:
  - Mexico City
  - Monterrey
  - Quintana Roo
History of modelling in Brazil

• Aristides Barreto proposes modelling for teaching (about 45 years ago).
• ICME-1 (1969) — Lyon (France)
• ICME-4 (1980) — Berkeley (USA)


http://www.furb.br/cremm/ingles/index.php
History of modelling in Brazil

- 1980 **Rodney Bassanezi** follows his lead with great strength.

- 1986 **Maria Salett Biembengut**, joins the quest.
Modelling in the curriculum

- Paraná State integrated modelling into the curriculum.
- Other states followed Paraná.
- Then, modelling gets integrated into the national curriculum for all grade levels.

- Moreover, the national high school exam (ENEM) integrates modelling into the assessment.
## PISA Results 2012

<table>
<thead>
<tr>
<th>Country</th>
<th>Mean score</th>
<th>Low achievers (Below Level 2)</th>
<th>Top performers (Level 5 or 6)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Chile</td>
<td>423</td>
<td>51.5</td>
<td>1.6</td>
</tr>
<tr>
<td>Mexico</td>
<td>413</td>
<td>54.7</td>
<td>0.6</td>
</tr>
<tr>
<td>Uruguay</td>
<td>409</td>
<td>55.8</td>
<td>1.4</td>
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<tr>
<td>Costa Rica</td>
<td>407</td>
<td>59.9</td>
<td>0.6</td>
</tr>
<tr>
<td>Brazil</td>
<td>391</td>
<td>67.1</td>
<td>0.8</td>
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<tr>
<td>Argentina</td>
<td>338</td>
<td>66.5</td>
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<tr>
<td>Colombia</td>
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<td>73.8</td>
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<tr>
<td>Perú</td>
<td>368</td>
<td>74.6</td>
<td>0.6</td>
</tr>
</tbody>
</table>
The Case of South Africa
The role of modelling in the South African curriculum

Helena Wessels, Stellenbosch University
Intended school curriculum

• **Primary school**
  • Modelling not explicitly mentioned, problem solving a focal point
  • General and specific aims are
    • to identify and solve problems, making decisions using critical and creative thinking
    • to demonstrate understanding that the world is a set of related systems and that problem solving contexts do not exist in isolation
    • Cognitive development (problem solving, logical thought & reasoning)
  • Non-routine problems, higher-order understanding and processes, breaking up problems in constituent parts emphasized for upper primary school
  • Assessment of problem solving skills described in curriculum – example rubrics and scoring given
Intended school curriculum

• High school
  • General and specific aims are
    • to identify and solve problems, making decisions using critical and creative thinking
    • to demonstrate understanding that the world is a set of related systems and that problem solving contexts do not exist in isolation
    • Cognitive development (problem solving, logical thought & reasoning)
  • Mathematical modeling as focal point: real-life problems should be included, realistic and not contrived. Contexts should include health, social, economic, cultural, scientific, political and environmental issues
  • Non-routine problems, higher order reasoning & processes emphasised
  • Assessment:
    • “Modelling as process should be included in all papers, thus contextual questions can be set on any topic”
    • Memo with model answers
    • Department of Basic Education discourages the use of rubrics
Enacted school curriculum

- Pressure of Annual National Assessments – little time for real problem solving or modelling
- Teachers’ understanding of problem solving → textbook word problems
- Teachers’ understanding of modelling and modelling problems rudimentary or non-existent
- Many high school teachers use textbook problems to comply
Tertiary education

• Modelling courses or units in undergraduate and honours courses at a number of SA universities

• Assessment tasks; peer and lecturer assessment, group assessment

• Modelling topics in M and PhD studies at very few universities
ICTMA role?

• Attending of ICTMA conferences, discussions with cognoscenti who also attended, joint projects with colleagues from other universities internationally, fostered the development and improvement of modelling at some of the universities and schools in SA

• Bank with modelling problems for all grades needed – ICTMA members contribute (coordinated by ICTMA special committee??)
The Case of Australia
Mathematical Modelling and Applications in Australia

Vince Geiger
Australian Catholic University
Curriculum Landscape
Curriculum Landscape
Curriculum Landscape
Curriculum Landscape
Curriculum Landscape
That all systems and schools recognise that, while mathematics can be taught in the context of mathematics lessons, the development of numeracy requires experience in the use of mathematics beyond the mathematics classroom, and hence requires an across the curriculum commitment.

(Council of Australian Governments, 2008, p. 7)
Numeracy as a General Capability

Using mathematical skills across the curriculum both enriches the study of other learning areas and contributes to the development of a broader and deeper understanding of numeracy. Therefore, a commitment to numeracy development is an essential component of learning areas across the curriculum and a responsibility for all teachers. This requires that teachers:

- identify the specific numeracy demands of their learning area
- provide learning experiences and opportunities that support the application of students’ general mathematical knowledge and skills
- use the language of numeracy in their teaching as appropriate.
Students become numerate as they develop the knowledge and skills to use mathematics confidently across all learning areas at school and in their lives more broadly. Numeracy involves students in recognising and understanding the role of mathematics in the world and having the dispositions and capacities to use mathematical knowledge and skills purposefully.

Students develop numeracy capability as they learn to organise and interpret historical events and developments. Students learn to analyse numerical data to make meaning of the past, for example to understand cause and effect, and continuity and change. Students learn to use scaled timelines, including those involving negative and positive numbers, as well as calendars and dates to recall information on topics of historical significance and to illustrate the passing of time.
Numeracy in the Australian Professional Standards for Teachers

- Standard 2.5 Literacy and numeracy strategies

Graduate career stage: Know and understand literacy and numeracy teaching strategies and their application in teaching areas.

Proficient career stage: Apply knowledge and understanding of effective teaching strategies to support students’ literacy and numeracy achievement.
Very little mention of modelling (or numeracy) throughout the document but...

Three out of four senior mathematics subjects refer to modelling in the relevant assessment standards e.g.,

**Specialist mathematics**

*Concepts and Techniques*
develops, selects and applies mathematical models to routine and non-routine problems in a variety of contexts

*Reasoning and communication*
identifies and explains the validity and limitations of models used when developing solutions to routine and non-routine problems
Modelling in Queensland

- Syllabus objective Mathematics B
  1. Knowledge and procedures
  2. Modelling and problem solving
  3. Communication and justification
Modelling in Queensland

**Modelling and problem solving**

The objectives of this category involve the uses of mathematics in which the students will model mathematical situations and constructs, solve problems and investigate situations mathematically across the range of subject matter in this syllabus. By the end of the course students should be able to:

- apply problem-solving strategies and procedures to identify problems to be solved, and interpret, clarify and analyse problems
- identify assumptions (and associated effects), parameters and/or variables during problem solving
- represent situations by using data to synthesise mathematical models and generate data from mathematical models
- analyse and interpret results in the context of problems to investigate the validity (including strengths and limitations) of mathematical arguments and models.
Modelling in Queensland

Principles of a balanced course

- Application
- Technology
- Initiative
- Complexity
Maths in Action: Applications and Modelling Community

- Mathematica, computer-based maths and the new era of STEM
- Pulsars in the mathematics classroom
- What's happened to temperatures over the last century?
- Mathematica and the Barn Quilt Project
- Discovering Sustainability & Maths in a World Heritage Icon
The Cases of Netherlands & Norway
Modelling in the Dutch and Norwegian Curriculum

Pauline Vos, Agder University
What is the role of modelling in the curriculum of your country, including assessment?

- Realistic Mathematics Education was “born” in NL:
  - Realistic contexts to start from "mathematics as to be useful" (Hans Freudenthal)
    usefulness experienced during the learning process
  - Modelling as a vehicle for concept development
- Occasionally: modelling-for-modelling (day-long projects, A-lympiad)
- National exams: 100% of tasks is contextualised (exception: MathB 58-74%), 9-12 pages of text, “reproductive mathematizing”, ready-made models, never a real problem being solved
With what % does the value decrease each year?

Show that the value after 5 years is 40 euro.

Celise gaat met de fiets naar haar werk. Ze heeft hiervoor een nieuwe fiets gekocht van € 530,-. Celise is van plan haar fiets na een aantal jaren in te ruilen. De fiets wordt elk jaar minder waard. Ze gebruikt de volgende formule als vuistregel voor het berekenen van de inruilwaarde:

$$w = 530 \times 0,6^t$$

Hierin is $w$ de inruilwaarde van de fiets in euro en $t$ het aantal jaren dat de fiets oud is.

Bij de vragen 17, 18 en 19 gaan we uit van bovenstaande formule.

17 → Met hoeveel procent neemt de inruilwaarde van haar fiets elk jaar af?

18 → Celise wil weten wat de inruilwaarde van haar nieuwe fiets na 5 jaar zal zijn. Laat zien dat de nieuwe fiets van Celise na 5 jaar ongeveer € 40,- waard is. Schrijf je berekening op.
Calculate \( a \) and \( b \) for the formula.

<table>
<thead>
<tr>
<th>( h )</th>
<th>10</th>
<th>20</th>
<th>30</th>
<th>40</th>
<th>50</th>
<th>60</th>
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<tr>
<td>( W )</td>
<td>1,2</td>
<td>1,6</td>
<td>2,1</td>
<td>2,5</td>
<td>3,0</td>
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<td>3,9</td>
<td>4,3</td>
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</table>

Bereken \( a \) en \( b \) met behulp van de gegevens in tabel 1. Rond \( a \) af op drie decimalen en \( b \) op twee decimalen.

Onderzoek door weerkundigen naar windsnelheden op verschillende hoogtes en onder verschillende omstandigheden heeft opgeleverd dat het verband tussen windsnelheid en hoogte in het algemeen niet lineair is. Een betere formule is:

\[
W = 5,76 \cdot m \cdot \log \left( \frac{h}{r} \right)
\]

Hierin is:
- \( W \) de windsnelheid (in m/s);
- \( h \) de hoogte in meter waarop de windsnelheid wordt gemeten;
- \( m \) een constante die afhankt van de wrijving tussen de luchtlagen;
- \( r \) een constante die afhankt van de ruwheid van het terrein (hoge bomen beïnvloeden de windsnelheid anders dan grasland)

De formule is geldig tot hoogtes van ongeveer 100 meter.

In de praktijk wordt de windsnelheid op een hoogte van 10 meter gemeten. De waarde van \( r \) op de meetplek is bekend zodat het getal \( m \) met behulp van de formule berekend kan worden. Vervolgens kan met de gegeven formule de windsnelheid op andere hoogtes berekend worden.

Boven open bouwland met \( r = 0,12 \) wordt de windsnelheid gemeten. Op 10 meter hoogte is deze windsnelheid 6,0 m/s.

Bereken in deze situatie de windsnelheid op een hoogte van 60 meter.
The distance between A and B at time $t$ is given by $a(t) = \ldots$

Show this.
Are there big discrepancies to actual classroom practice?

- What is “big”?

Not big:
- Assessment is driving
- Many teachers attend “NWD” (a fair on math applications)

Yes, big:
- Teachers’ dependency on textbooks
- Mechanistic modelling (drill of atomistic modelling)
- Math time being cut (time tables)
What role - if any - has the international modelling community played in the developments in your country, and which role could it play?

- The other way around: the role of Dutch leaders in the international community
  Internationally frequently cited:
  Hans Freudenthal, Jan de Lange, Marja van den Heuvel-Panhuizen, Koen Gravemeijer and others

- many design studies
  Adri Dierdorp (2012) - Learning statistical modelling based on authentic practices

- few evidence-based evaluations, few cognitive studies
What is the role of modelling in the curriculum of your country, including assessment?

• Virtually inexistent in curriculum documents (terms used: relevance, engaging, attractive, inquiry-based)

• National exams consist of 2 parts
  1\textsuperscript{st} part: without tools, classical “barren” maths
  2\textsuperscript{nd} part: with tools (calc, CAS, off-line computer) more contextualised, never a real problem being solved
When will the saturation be more than 0.8?

Use the derivative to show that the saturation rises when the pressure rises.

Oppgave 2 (6 poeng)

I lungene blir oksygen bundet til hemoglobin og transportert rundt i kroppen av blodet. Hemoglobiniet er mettet når det ikke er i stand til å ta opp mer oksygen.

Den engelske fysiologen A. V. Hill oppdaget i 1910 en sammenheng mellom deltrykket til oksygenet i lungene og metningsgraden \( g \).

Han fant at under visse forhold er

\[
g(x) = \frac{x^3}{x^3 + 25000}, \quad x > 0
\]

er er deltrykket \( x \) målt i mmHg (millimeter kvikksølv).

a) Bruk graftegner til å tegne grafen til \( g \).

b) Bestem grafisk hva deltrykket \( x \) må være for at metningsgraden \( g(x) \) skal være større enn 0.8.

c) Bruk derivasjon til å vise at metningsgraden øker dersom deltrykket øker. Forklar.
Use regression to create the models $A(x)$ and $B(x)$ for the relation between windspeed and effect.

For which wind speed have the windmills the same effect?
Are there big discrepancies to actual classroom practice?

• What is “big”?

• Teachers and students prefer to prepare for the 1\textsuperscript{st} part of the exams

• 2\textsuperscript{nd} part of exams is relatively new, teachers not at ease with use of CAS
What role - if any - has the international modelling community played in the developments in your country, and which role could it play?

• **Staff mobility within Nordic countries**
  for example
  Morten Blomhøj – Univ of Tromsø
  Mette Andresen – Univ of Bergen
  Mogens Niss – Univ of Agder project

• **Seminars with international experts**
  2914 Seminar on Vocational Education with G Wake, R Sträßer

• **A need for research**
  Suela Kacerja (2012) - ”Real-life contexts in mathematics and students’ interests. An Albanian study”
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<th>Contextualised</th>
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