ICTMA Newsletter

Volume 9 Number 1 June 2016

The International Community of Teachers of Mathematical Modelling and Applications.
www.ictma15.edu.au

The Community, through its membership, research and other activities, is recognised as "The International Study Group for Mathematical Modelling and Applications (ICTMA)" by its affiliation to the International Commission on Mathematical Instruction (ICMI).

Editor	*President*
Associate Professor Gloria STILLMAN Faculty of Education and Arts Ballarat Campus Australian Catholic University, AUSTRALIA Email: gloria.stillman@acu.edu.au	Associate Professor Gloria STILLMAN Faculty of Education and Arts Ballarat Campus Australian Catholic University, AUSTRALIA Email: gloria.stillman@acu.edu.au

Contents

Please send future contributions to the editor by email <gloria.stillman@acu.edu.au>. The next Newsletter will be published in December, 2016. We are interested in your contributions to any of the current sections as well as project reports and modelling problems.

Website

Owing to government action beyond our control the ICTMA website was removed. Please report broken links and send contributions to Jill Brown jill.brown@acu.edu.au The site is currently working through the former site for ICTMA15. Please use the URL above until further notice.

1. International Executive Committee

Following the business meeting of ICTMA members held at ICTMA 17 in Nottingham, England, on July 24, 2015, the ICTMA Executive for 2015-2017 was confirmed as follows:

President

A/Prof Gloria Stillman (Australia) – ***Newsletter Editor*** [Email: gloria.stillman@acu.edu.au]

Elected Members

Dr Jill Brown (Australia) – ***Secretary, Webmaster & List Serve Moderator*** [Email: Jill.Brown@acu.edu.au]

A/Prof Susana Carreira (Portugal) – [Email: scarrei@ualg.pt]

Prof Dr Gabriele Kaiser (Germany) [Email: gabriele.kaiser@uni-hamburg.de]

Co-opted Members

A/Prof Angeles Dominguez (Mexico) [Email: angeles.dominguez@itesm.mx]

Prof Toshikazu Ikeda (Japan) – ***Registrar*** [Email: ikeda@ynu.ac.jp]

Prof Jinxing Xie (China) [Email: jxie@math.tsinghua.edu.cn]

Conference Organisers

Prof Maria Salett Biembengut (Brazil) [Email: mariasalettb@gmail.com]

A/Prof Geoff Wake (UK) [Email: Geoffrey.Wake@nottingham.ac.uk]

Dr Helena Wessels (South Africa) [Email: hwessels@sun.ac.za]

2. Forthcoming Conference – ICTMA 18

18TH INTERNATIONAL CONFERENCE ON THE TEACHING OF MATHEMATICAL MODELLING AND APPLICATIONS (ICTMA18) 23-28 JULY, 2017

Theme: Mathematical Modelling and Sense Making

Dr Helena Wessels, Chair ICTMA18

The Research Unit for Mathematics Education at the University of Stellenbosch (RUMEUS), Stellenbosch, South Africa, supported by the Mathematics and Mathematics Education bodies in South Africa (SANCIMU, SAMF, SAMS and AMESA), has been selected to host the next International Conference for the Teaching of Mathematics and Applications (ICTMA 18) from 23-28 July 2017. The Steering Committee consists of members of RUMEUS, that is Dr Helena Wessels (Member and Conference Chair, ICTMA18), Prof Dirk Wessels (Director), Dr Erna Lampen, and Dr Faaiz Gierdien, as well as Prof Mdu Ndlovu and Mr Cerenus Pfeiffer from the Centre for Pedagogy (SUNCEP) as members.

Plenary Speakers

Plenary speakers for the conference have been confirmed as:

Prof Alan Schoenfeld (University of Berkely, USA),

A/Prof Gloria Stillman (Australian Catholic University),

Prof Cyril Julie (University of the Western Cape, South Africa),

and Dr Piera Biccard (University of South Africa).

Visas

Visas are issued by the South African missions abroad and must be affixed in the applicant's passport before departing for South Africa. Visas are not issued on arrival at South African ports on entry. Many nationalities do not require a visa to enter South Africa, it is best to check with your travel agents if this is required. For detailed information, please visit:

http://www.services.gov.za/services/content/Home/ServicesforForeignNationals/Temporaryresidence/Applicationforavisa/en_ZA

International visitors to South Africa should know that there are special requirements for the visas of young children, even if accompanied by their parents. Please study the above website.

Currency

The Rand is the official currency of South Africa.

Travel

Cape Town International Airport is served by more than 20 international airlines on a weekly basis, linking the destination to global hubs like Frankfurt, Amsterdam, Dubai, Singapore and London (and a wide variety of other connections via Johannesburg). The city is an overnight flight from any European destination.

Further Information

The conference website is expected to go live during July 2016. For further information contact the conference chair, Dr Helena Wessels <hwessels@sun.ac.za>

Local Organising Committee Members

Prof Dirk Wessels

Dr Erna Lampen

Dr Faaiz Gierdien

Prof Mdu Ndlovu

Mr Cerenus Pfeiffer

Helena Wessels

3. Call for Bids to hold Upcoming Conference – ICTMA 19

19TH INTERNATIONAL CONFERENCE ON THE TEACHING OF MATHEMATICAL MODELLING AND APPLICATIONS (ICTMA19) 2019

Any groups considering hosting ICTMA 19 in 2019 are invited to send a written Expression of Interest to the President who will then send details of the requirements for a full bid. Full bids to host the conference in the form of a bidding book and supporting letters will be due in June 2017. It is best though to start working well before this to make a comprehensive proposal. Contact: Gloria Stillman via email gloria.stillman@acu.edu.au

4. Modelling at ICME 13: German Thematic Afternoon, Topic Study Group and Affiliated Study Group Meetings

ICTMA members at ICME 13 in Hamburg have a wealth of choices of presentations to attend on Mathematical Modelling at the Congress. Many of our members are involved in these and, of course, Gabriele Kaiser has been doing a stellar job as the Conference Convenor. Please support these meetings and events.

4.1 Thematic Afternoon: Mathematical Modelling in German Speaking Countries

There is an excellent afternoon of presentations proposed for this thematic afternoon that would benefit by your participation in discussions and support. Please consult the ICME website for full details.


The planned program for the afternoon of Wednesday 27 July is as follows:

16:30: *Mathematical Modelling in German Speaking Countries: Introduction and Overview* (Gilbert Greefrath)

16:50: Cognitive and empirical approaches
Classification of Modelling Cycles – a view insight cognitive processes (Rita Borromeo Ferri)
Quantitative Research on Modelling – Examples from German Speaking Countries (Dominic Leiss, Stanislaw Schukajlow)

17:30: Promoting modelling competencies
Mathematical Modelling in professional development – traditions in Germany (Katja Maaß)
Implementing mathematical modelling in schools (Katrin Vorhölter)

18:10-18:30: Closing
International Perspective on the German modelling debate (Gloria Stillman)
Discussion (Katrin Vorhölter)

4.2 Topic Study Group Meetings at ICME 13

TSG21 at ICME 13 will focus on Mathematical Applications and Modelling in the Teaching and Learning of Mathematics. The TSG has been well supported by full papers, orals and posters showcasing research into mathematical modelling education. The planned structure of the main TSG sessions involves invited plenaries and 20 minute research and/or practice presentations. As well as this there are parallel oral communication sessions. The following was the program at time of writing but please visit TSG21 on the ICME 13 website at www.icme13.org for updates.


Tuesday 26 July: 12.00-13.30 Chair Jussara Araújo

12.00-12.30: Welcome and overview of sessions
12.30-13.30: Plenary – *State of the Art on Modelling in Mathematics Education* (Gloria Stillman)

Oral Communications in three parallel sessions 15:00-18:00 see ICME 13 website for details


Wednesday 27 July 12.00 – 13.30 Two parallel sessions of full paper presentations

Session 1 Chair: Morten Blomhoj
France Caron – *Approaches to investigating complex dynamical systems*
Irit Peled – *Shifts in knowledge and participation of children with mathematical difficulties working on modelling tasks*
Dung Tran – *Authenticity of modeling tasks and students' problem solving*

Miriam Ortega – *Influence of technology on mathematical modelling of a physical phenomenon*

Session 2 Chair: Toshi Ikeda
Takashi Kawakami - *Merging of task contexts and mathematics in dual modelling teaching: Case studies in Japan and Australia*
Jill P Brown – *What do we mean by 'context'?*
Andreas Busse – *The negative impact of the new German examination tasks on the modelling classroom in Hamburg*
Corinna Hertleif – *Assessing sub-competencies of mathematical modelling in the LIMO project*


Friday 29 July 12.00 – 13.30 Two parallel sessions of full paper presentations

Session 1: Chair Dominik Leiss
Xenia Reit – *The potential of cognitive structures in solution approaches of modelling tasks*
Jennifer Czocher – *Making sense of student-generated conditions and assumptions*
Angles Dominguez – *Model application activity: Integration of concepts and models*
Toshi Ikeda - *Organizing mathematical modelling in Japanese mathematics curriculum*

Session 2: Chair Jill P. Brown
Juhaina Awawdeh Shahbari – *Adapting a cognitive tool for representing teachers' interpretations of students' modeling activities*
Peter Stender – *Heuristic strategies in modelling problems*
Elizabeth W. Fulton – *Teachers as learners: Understanding and valuing mathematicsl modelling through professional development*
Joo Young Park – *Pre-service mathematics teachers project-based mathematical modeling instruction: Conception, task design, and enactment*

Oral Communications – Parallel sessions 15:00-18:00 see ICME 13 website for details


Saturday 30 August - Chair: Gloria Stillman

12.00-12.30: Plenary – *Toward a Framework for a Dialectical Relationship between Pedagogical Practice and Research* (Jussara Araújo)
12.30-13.30: Plenary – *Interplay between Research and Development of Teaching Practices in Mathematical Modelling* (Morten Blomhoj)
13.00-13.30: Summary and closing

4.3 Affiliated Study Group Meetings at ICME 13

As many of you are involved in other TSGs the ASG meetings will be a chance to catch up with ICTMA friends.


There is one session on Saturday 30th July.

Planned structure:

16:30- 16:40: Introduction and Welcome by President – Gloria Stillman

16:40-17:10: *Modelling for the 21st Century - Henry Pollak*

17:10-17:35: *Modelling Challenges and other activities – Jill Brown*

17:35-17:55: *Next ICTMA Conference ICTMA18 in South Africa – Helena Wessels*

17:55-18:00: Thank you and Farewell by President – Gloria Stillman

5. Brief News Items

5.1 Results of Second International Modelling Challenge IM^{2}C

IM^{2}C is a mathematical modelling competition for teams of up to four students from secondary schools around the world that seeks to develop and enhance students' ability to visualise, understand and apply mathematics in the development of an original mathematical model to solve a common problem. There were 23 countries/regions invited to participate in the 2016 IM^{2}C and after the national selection round, 40 teams competed in the international round of judging. All teams worked at their own schools during a 5 day period between March 16th and May 9th, 2016. Each team was given a modelling problem and then constructed their solutions. This year's problem, Record Insurance, asked the teams to design a model for the effective planning for payments and insurance for record-breaking performances at a track and field meet.

All schools were to be commended for their efforts. The judges were impressed with all the teams' creativity and ingenuity in mathematical modelling and in their ability to explain their strategies and problem-solving techniques in clear terms. Initial funding for planning and organizational activities was provided by IM^{2}C co-founders and co-sponsors The Consortium for Mathematics and its Applications (COMAP), a not-for-profit company dedicated to the improvement of mathematics education and by NeoUnion ESC Organization in Hong Kong.

The three teams judged as outstanding were:

Palo Alto High School, Palo Alto, California,USA, (Advisor, Radu Toma), team members: Eric Foster, Kathryn Li, Kangrong Zhang and Andrew Lee
Diocesan Girls' School, Kowloon, Hong Kong (SAR), (Advisor, Yeung Po Ki), Team members: Cheng Wai Chung, Liang Hui Lin, Jia Jimsyn, and Poon Ho Kiu Allie
Pui Ching Middle School, Kowloon, Hong Kong (SAR), (Advisor, Lee Ho Fung), Team members: Wong Tsz Chun, Ling Janice, Ngai Chi Ki, and Lynn Shung Hei

A complete results report listing all teams as well as the full problem statement can be found at www.immchallenge.org. For additional contest information, contact IM^{2}C at: info@immchallenge.org.

Ross Turner

5.2 Springer Annual Book Reports for Books in ICTMA Series

Books in the Springer/ICTMA series, *International Perspectives on the Teaching and Learning of Mathematical Modelling*, show very strong impact worldwide as indicated by the Annual Book Performance Reports for 2015 recently issued by Springer to the editors. The latest 3 books in the series were in **the top 25% most downloaded eBooks** in the relevant Springer eBook Collection in 2015**.**

Trends in Teaching and Learning of Mathematical Modelling (68 chapters) published in 2011 has 60,297 chapter downloads. *Teaching Mathematical Modelling: Connecting to Research and Practice* (52 chapters), published late in 2013, has 49,625 combined chapter downloads. The latest volume in the ICTMA series, *Mathematical Modelling in Education Research and Practice: Cultural, Social and Cognitive Influences* (50 chapters) published in the second half of 2015 has 8,544 combined chapter downloads. The latest Bookmetrix data are available to everyone from www.bookmetrix.com or the publisher's website where the panel on the right hand side has a section called Book Metrics.

5.3 14th International Conference of the Mathematics Education for the Future Project

The 13th International Conference of the Mathematics Education for the Future Project in Catania, Sicily September 2015, was attended by 130 people from 22 countries. The next conference will be held next year at Balatonfüred, Balaton lake, Hungary from September 10-15, 2017. The conference title, *Mathematics Education for the next Decade*, continues our search for innovation in mathematics, science, computing and statistics education. Our thirteen previous conferences since 1999 were renowned for their friendly and productive atmosphere, and attracted many *movers and shakers* from around the world. We now call for papers and workshop summaries for presentation at the conference and publication in the printed conference proceedings. For further details and updates please email alan@cdnalma.poznan.pl

Alan Rogerson

6. News from the Regions

6.1 Australia's First IM²C for School Students a Success

Entry of Australia into the International Mathematical Modelling Challenge (IM²C) has been s resounding success. The solutions of Australia's winning teams show that mathematical thinking is all around us. IM²C was coordinated in Australia by the Australian Council for Educational Research (ACER). Several Australian ICTMA members lent their support to this project. The problem set for the 2016 Challenge asked students to investigate how the organisers of an athletics competition could minimise their financial risk as they considered offering incentives to attract top-level competitors. Australia's best two solutions received Meritorious Awards, the second highest award category, in the international component of the competition. The two teams, Perth Modern School and Trinity College, Perth, were among 24 Australian teams who unpacked the problem over five consecutive days in order to develop a working solution before submitting a report to the Australian judging panel.

Ross Turner, project director for IM²C at ACER, commended the Perth Modern School and Trinity College teams for their international achievement, and the mathematical modelling work by all 14 teams who submitted entries to the national phase of the competition. "Mathematical modelling helps students to see the usefulness of mathematics," Mr Turner said. "It can help to build a bridge across what is often a yawning chasm between the mathematical work students typically do in school and the ways in which mathematics can be used to deal with important challenges in different real-world contexts. Mathematical modelling is used in many walks of life to employ mathematical tools and knowledge to describe and analyse situations in the real world. While many in the science, technology, engineering and mathematics (STEM) fields conduct their work by developing and using mathematical models, mathematical modelling is not something that only high-powered STEM professionals do. Many everyday applications of mathematics undertaken by all of us, every day, from revising a recipe to feed more people to figuring out a more efficient route to work, are instances of mathematical modelling in action."

Australia's outstanding IM²C team members are Alan Cheng, Virinchi Rallabhandi, Alex Rohl and Daniel Ho from Perth Modern School, and Samuel Carbone, Kayvan Gharbi, Farruh Mavlonov and Trong Nguyen from Trinity College, Perth. Other award-winning school teams in the national phase of the IM²C were from Canberra Grammar School; Coomera Anglican College, Qld; Glen Waverley Secondary College, Melbourne; Manea Senior College, Bunbury, WA; Mildura Senior College, Vic; Ormiston College, Brisbane; Somerville House, Brisbane; and St Ursula's College, Toowoomba, Qld. Registrations for the 2017 IM²C are now open. For more information, visit www.immchallenge.org.au

7. Recent Dissertations

Reit, X-R. (2015). Denkstrukturen in Lö sungsansätzen von Modellierungsaufgaben. Doctoral thesis. Johann Wolfgang Goethe University, Frankfurt on Main. Supervisors: Prof. Dr. Uwe Brinkschutle, Prof. Dr. Reinhard Oldenburg & Prof. Dr. Gilbert Greefrath. The dissertation has been published as follows: **Reit, X-R. (2016). Denkstrukturen in Lösungsansätzen von Modellierungsaufgaben: Eine kognitionspsychologische Analyse schwierigkeitsgenerierender Aspekte. Springer Fachmedien Wiesbaden. DOI 10.1007/978-3-658-13189-0**

Mathematical modelling as a process in which mathematics is used to elaborate a realistic problem is named in the general competencies to be fostered by school curricula (e.g., the German education standards for secondary education) and in the framework for international comparative studies such as PISA. However, the proportion of mathematical modelling in school practice is low. The disparity between educational requirements and everyday teaching in schools shows there is a need for action. This thesis contributes to this area by working out difficulty-generating aspects based on actual student solutions, particularly from a cognitive psychology perspective.

Initially, student solutions related on one aspect such as the same mathematical model were clustered into solution approaches. The cognitive structure of solution approaches was then analysed for parallel and sequential thought operations. A hypothesis derived from cognitive psychology is that the number of parallel thought operations has an influence on the difficulty of the respective solution approach. The aim was to develop a valid method to determine the theoretical difficulty of solution approaches. In addition, an assessment scheme was developed which is based on the cognitive psychological approach used for the determination of difficulty. By comparison of theoretical difficulty and average score of the respective solution approach as a measure for the empirical difficulty, a statement can be made about the validity of different aspects of the developed method for the determination of difficulty. An extension to complete modelling tasks resulted in a description of a theoretical task difficulty.

Five modelling tasks were developed for the study. Test booklets contained three tasks each which were solved by approximately 600 Year 9 students (15-16 years old). Approximately 1800 solutions were available for examination of difficulty. Solutions were classified into different approaches and scored using a scheme developed for the study. The average score of a solution approach as an indicator of empirical difficulty was compared with the theoretical difficulty as described above.

Statistical analysis confirmed that thought operations being processed in parallel in a solution approach led to a complication of that approach. With respect to complete modelling tasks, there was a similar relation between parallel thought operations and difficulty, but it was less dominant. As a result, the findings support a mostly intuitive approach by teachers in developing an assessment scheme for mathematics tasks in identifying and scoring important immediate steps which are then summated to a total score. As these results confirm the complication by parallelism of thought operations in task difficulty, the potential of cognitive psychology based methods in researching task difficulty and assessment in modelling tasks is confirmed. The study also contributes to our understanding of the complex structure of student solutions in modelling tasks. Such understanding is an important progression in understanding teacher management of modelling tasks and their integration into everyday school practice.

Stender, P. (2016). Wirkungsvolle Lehrerinterventionsformen bei komplexen Modellierungsaufgaben. Doctoral thesis. University of Hamburg. Supervisor: Prof. Dr. Gabriele Kaiser. The dissertation has been published as follows: **Stender, P. (2016). Wirkungsvolle Lehrerinterventionsformen bei komplexen Modellierungsaufgaben. Perspektiven der Mathematikdidaktik. Springer Fachmedien Wiesbaden. DOI 10.1007/978-3-658-14297-1**

This study examined how teachers should act when mentoring students working on a complex modelling problem. On the one hand, students should act as independently as possible working on modelling problems but on the other hand a meaningful solution needs to be created. The work of Aebli (1961), Zech (1998), Leiss (2007) and van de Pol, Volman and Beishuizen (2010) was a basis

for theoretical concepts in teacher intervention for the study whilst modelling theory was based on the current state of research shown by such work as Kaiser, Blum, Borromeo-Ferri and Greefrath (2014).

The research was conducted during three modelling days that took place at a German higher-level secondary school. One hundred and sixty Year 9 students worked on one of three modelling problems presented to them. Ten student groups were videotaped over the duration of the study. These students worked on the problem: Which intersection design (roundabout or traffic light) allows more cars to pass? Two tutors supervised these videotaped groups. Additional studies were carried out with experienced teachers at the same school who had undergone training.

Qualitative content analysis following Mayring (2010) was used to analyse the data related to interventions by tutors and experienced teachers. Major findings included: Strategic interventions were used to a significant extent unlike in the Leiss (2007) study. These interventions were mostly successful. Tutors had different preferences for the length of their interventions and had a slight tendency to increase the intensity of interventions towards the end of the modelling days. Invasive interventions were unsuccessful when students were working but tutors used these mainly when students were not working. When students were working tutors mainly used responsive interventions. Inadequate understanding of both the modelling and mathematical situations by tutors led to misleading interventions. The teacher intervention, "Explain your work", was very effective and used frequently. Further findings from additional studies included: A modelling cycle was able to be successfully used as a structuring element in an intervention and also to stimulate single steps in modelling. Experienced teachers intervened more frequently than student tutors and their interventions were to a significant extent strategic. Interventions making use of heuristic strategies were rarely observed. As the use of heuristic strategies was considered a good approach to formulate strategic interventions and these fit well with the goal of scaffolding student autonomy, the possibilities of formulating strategies and teacher interventions using these were examined but the effectiveness of these interventions was not explored in this study.

References

Aebli, H. (1961). Grundformen des Lehrens: Eine allgemeine Didaktik auf kognitionspsychologischer Grundlage (1. Aufl.). Stuttgart: Klett-Cotta.

Kaiser, G., Blum, W., Borromeo Ferri, R., & Greefrath, G. (2014). Modellieren im Mathematikunterricht. In R. Bruder, L. Hefendehl-Hebecker, B. Schmidt_Thieme & H.G. Weigand (Eds.), *Handbuch der Mathematikdidaktik* (pp. 357-383)). Berlin: Springer.

Leiss, D. (2007). "Hilf mir es selbst zu tun": Lehrerinterventionen beim mathematischen Modellieren. Doctoral dissertation, University of Kassel.

Mayring, P. (2010). Qualitative Inhaltsanalyse: Grundlagen und Techniken. Weinheim und Bassel: Beltz Verlag.

van de Pol, J., Volman, M., & Beishuizen, J. (2010). Teacher scaffolding in small-group work: an intervention study. *Journal of the Learning Sciences, 23*(4), 600-650.

Zech, F. (1998). Mathematik erklären und verstehen: Eine Methodik des Mathematikunterrichts mit besonderer Berücksichtigung von lernschwachen Schülern und Alitagsnähe (2., durchges. Aufl.). Berlin: Cornelsen.

Tan, Liang Soon (2016). Professional Development for Teachers of Mathematical Modelling in Singapore. Doctoral thesis, National Institute of Education, Nanyang Technological University, Singapore. Supervisor: Prof. Ang Keng Cheng.

This thesis describes the conceptualisation, implementation, and evaluation of a *school-based* professional development (SBPD) program for teachers in mathematical modelling. The proposed SBPD program was new in the context of local schools in Singapore. It aimed to help novice teachers develop competencies to make appropriate decisions in the teaching of mathematical modelling. To evaluate the SBPD program, the study addressed the following research questions:

1. What aspects of a mathematics teacher's competencies in teaching mathematical modelling are developed through his/her participation in the SBPD program?

2. What are the reasons for the development (or the lack of development) of these aspects of a teacher's competencies to teach mathematical modelling in relation to the SBPD program?

A wide selection of relevant literature was reviewed to establish the theoretical framework within which the SBPD program was conceptualized. The SBPD program was characterized by three dimensions of learning deemed essential for effective professional development of teachers – *content*, *process* and *context*. The content dimension is focussed on developing the teacher's knowledge and skills in the teaching of modelling. The process dimension is characterized by the transformative learning cycles for teachers to elicit, enact and reorganize their orientations in a mathematical modelling classroom. The context dimension takes into account the school contextual factors and adjusts the SBPD programme implementation accordingly.

A multiple-case study approach was adopted. The SBPD program was implemented in three secondary schools in Singapore. Data collected across the classroom and SBPD program levels included lesson observations, formal interviews, teachers' commentaries, and analysis of relevant lesson artefacts of teachers' practice in the teaching of modelling. An analytic tool adapted from Schoenfeld's framework for goal-based decision-making (Schoenfeld, 2010) was used to examine participating teachers' orientations, resources, goals and teaching decisions.

The findings of this study indicate that the SBPD program positively influenced teachers' knowledge and resources, goals, and orientations in planning, designing, and enacting modelling learning experiences. Teachers were able to plan developmentally appropriate modelling lessons and design learning opportunities for students to reason mathematically their choices of modelling ideas and steps. Teachers moved from compartmentalized teaching of the modelling stages to facilitating students' learning experiences in the modelling process.

This study also reveals that the development of the teacher's competencies in the teaching of mathematical modelling had resulted from an interaction between the content and process dimensions of the SBPD program. A teacher's independent modelling learning experiences, knowledge of the modelling task solution space, and use of Ang's framework for teaching modelling (Ang, 2015) were necessary resources that were activated in the teaching of modelling. The teacher's experience in the transformative learning cycles has helped develop and reorganize orientations, resources and goals in the teaching of modelling.

In addition, teachers who participated in this study had begun to value students' mathematical reasoning during the modelling process and became more sensitised to their learning difficulties. They had internalized the use of Ang's framework for teaching modelling, developed more coherent lesson images and better questioning techniques. They also became oriented towards the learning goal of getting students to formulate the mathematical problem from the real world situation. The outcomes that were achieved suggest that the conceptualization of the SBPD program may be capable of wide and flexible applications.

References

Ang, K. C. (2015). Mathematical modelling in Singapore schools: A framework for instruction. In N. H. Lee & K. E. D. Ng (Eds.), *Mathematical modelling: From theory to practice* (pp. 57-72). Singapore: World Scientific.

Schoenfeld, A.H. (2010). How we think: A theory of goal-oriented decision making and its educational applications. New York, NY: Routledge.

8. Recent Publications of Interest

Bingölbali, E., & Coskun, M. (2016). A proposed conceptual framework for enhancing the use of making connections skill in mathematics teaching. *Education and Science, 41*(183), 233-249.

Carreira, S., Jones, K., Amado, N., Jacinto, H., & Nobre, S. (2016). Digitally expressing conceptual models of geometrical invariance. In *Youngsters solving mathematical problems with technology* (pp. 113-140). Cham, Switzerland: Springer.

Chong, M. S. F., & Shahrill, M. (2016). The use of an emerging framework to explore students' cognitive competency. *Indian Journal of Science and Technology, 9*(16), 1-12. Available from: www.indjst.org

Czocher, J. A. (2016). Introducing modeling transition diagrams as a tool to connect mathematical modeling to mathematical thinking. *Mathematical Thinking and Learning, 18*(2), 77-106.

Downs, N., Parisi, A.V., Galligan, L., Turner, J., Amar, A., King, R., Ultra, F., & Butler, H. (2016). Solar radiation and the UV index: An application of numerical integration, trigonometric functions, online education and the modeling process. *International Journal of Research in Education and Science (IJRES), 2*(1), 179-189.

English, L.D. (2016). Advancing mathematics education research within a STEM environment. In K. Makar, S. Dole, J. Visnovska, M. Goos, A. Bennison, & K. Fry (Eds.), *Research in mathematics education in Australasia 2012-2015* (pp. 353-371). Singapore: Springer.

Greefrath, G., & Vorhölter, K. (2016). *ICME-13 Topical survey --Teaching and learning mathematical modelling: Approaches and developments from German-speaking countries*. Springer.

Hirsch, C.R., & Roth McDuffie, A. (Eds.). (2016). *Annual perspectives on math education 2016: Mathematical modelling and modelling mathematics.* Reston, VA: National Council of Teachers of Mathematics.

Jung, H., & Brady, C. (2016). Roles of a teacher and researcher during in situ professional development around the implementation of mathematical modelling tasks. *Journal of Mathematics Teacher Education, 19*(2-3), 277-295.

Özkan Hıdıroğlu, Y., & Hıdıroğlu, Ç. N. (2016). Examining epistemological beliefs in explaining mathematics teachers' approaches in mathematical modelling. *Journal of Theory and Practice in Education, 12*(1), 17-40.

Nguyen, D.M. (2016). Modelling in Vietnamese school mathematics. *International Journal of Learning, Teaching and Educational Research, 15*(6), 114-126.

Siswono, Y.E.T., Kohar, A.W., Kurniasari, I., & Astuti, Y.P. (2016). An investigation of secondary teachers' understanding and belief on mathematical problem solving. *Journal of Physics: Conference Series 693* (2016) 012015.

Smith, C., & Morgan, C. (2016). Curricular orientations to real-world contexts in mathematics. *The Curriculum Journal, 27*(1), 24-45.

Stillman, G., Brown, J., Galbraith, P., & Ng, K.E.D. (2016). Research into mathematical applications and modelling. In K. Makar, S. Dole, J. Visnovska, M. Goos, A. Bennison, & K. Fry (Eds.), *Research in mathematics education in Australasia 2012-2015* (pp. 281-304). Singapore: Springer.

van Velzen, J. (2016). Evaluating the suitability of mathematical thinking problems for senior high-school students by including mathematical sense making and global planning. *The Curriculum Journal* DOI: 10.1080/09585176.2016.1174140

Vorhölter, K., & Kaiser, G. (2016). Theoretical and pedagogical considerations in promoting students' metacognitive modelling competencies. In C.R. Hirsch & A. Roth McDuffie (Eds.). (2016). *Annual perspectives on math education 2016: Mathematical modelling and modelling mathematics* (pp. 273-280). Reston, VA: National Council of Teachers of Mathematics.

Voskoglou, M.G., & Subbotin, I.Y. (2016). An application of fuzzy numbers to the assessment of mathematical modelling skills. *International Journal of Mathematical Modelling & Computations, 6*(1), 83-103.